
CHARMS: A Simple Framework for Adaptive Simulation

Eitan Grinspun

Petr Krysl

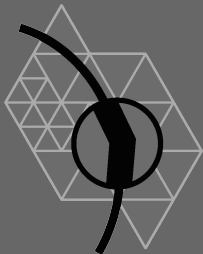
Peter

Schröder

Caltech

UCSD

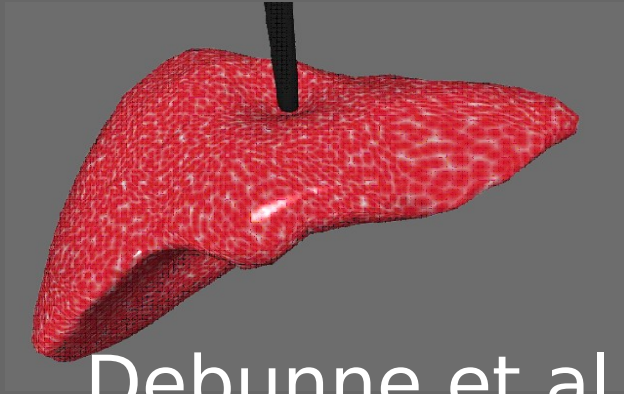
Caltech



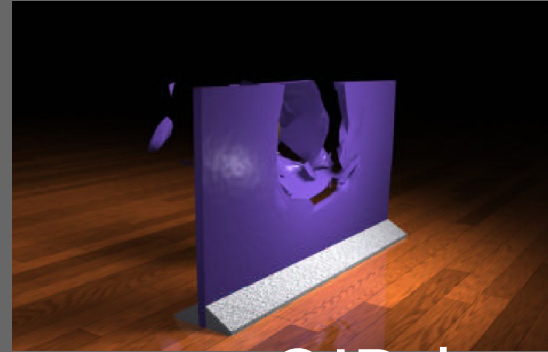
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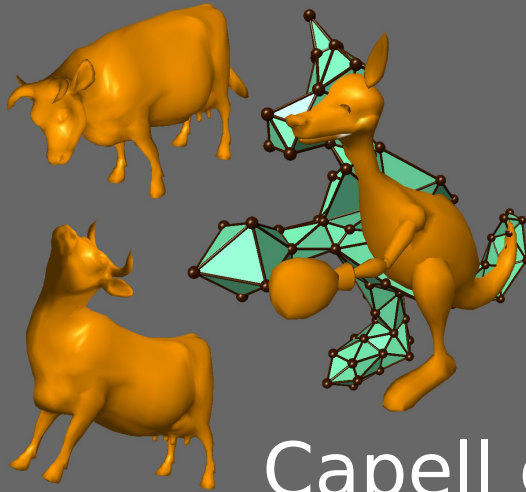
Context: Graphics & PDEs



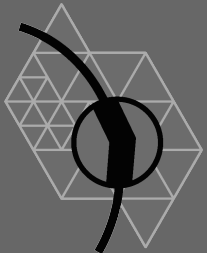
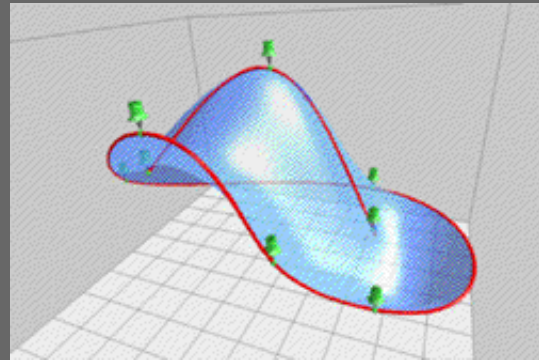
Debunne et al.



O'Brien et al.



Capell et al. Welch et al., Gortler et al.



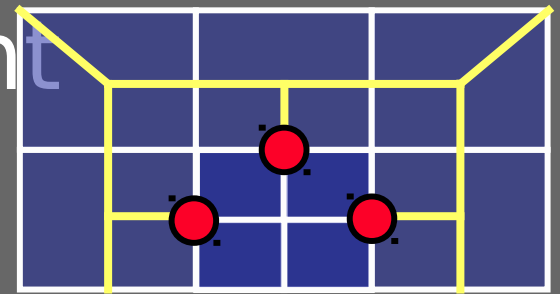
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Adaptivity Is Critical

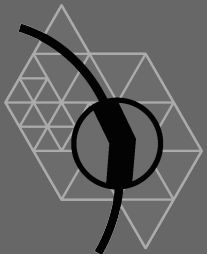
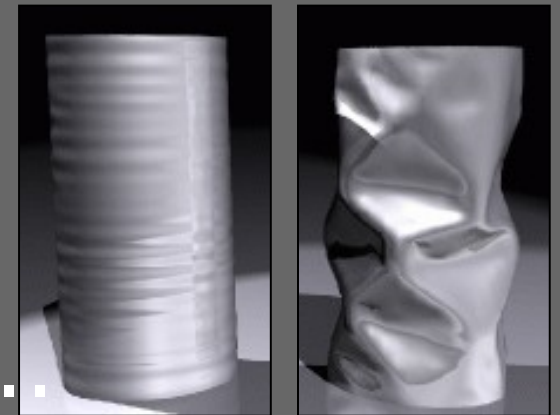
Adjust spatial resolution

- element refinement
- Cohen, Hodgins,...



Difficult...

- compatibility
- ...or unclear
- t-vertices, cracks
- subdiv., wavelets,...



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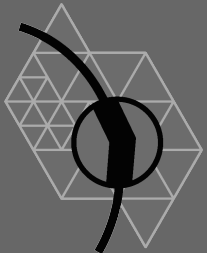
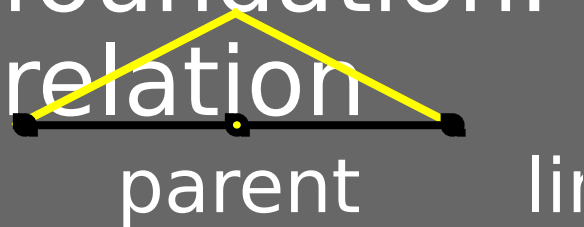
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Overview

Universal approach to
adaptivity

- indep. of: error
estimator/solver

- foundation:
relation



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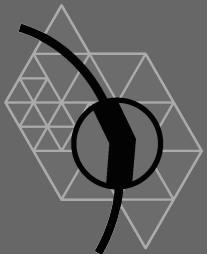
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Idea Takeaway

Conforming Hierarchical Adaptive
Refinement Methods

Element refinement considered harmful

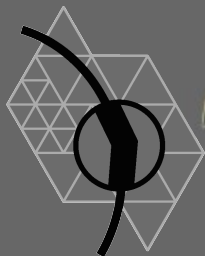
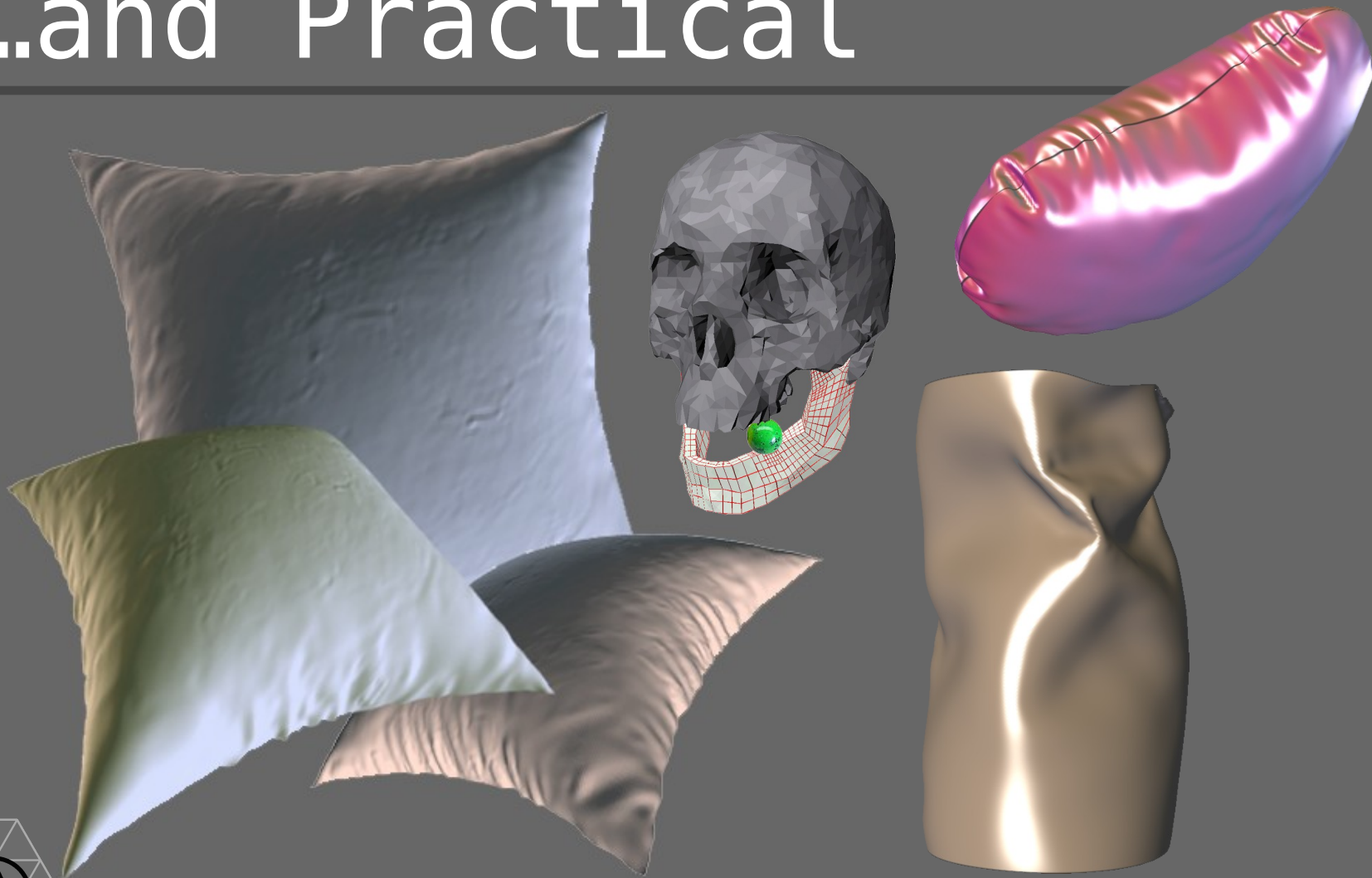
- refine basis functions, not elements
- simple (naturally compatible!)
- any dimension, order, element type
- unifies prior work



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...and Practical



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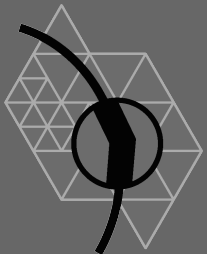
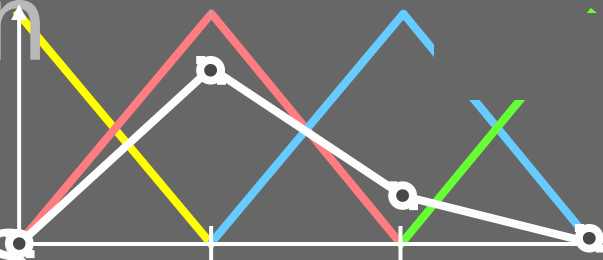
Setting

Example problem

- Laplace's eqn.

FE Discretization

- $u(x)$ built of basis functions
- linear system
- fn. overlaps



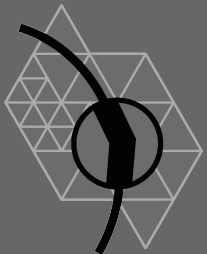
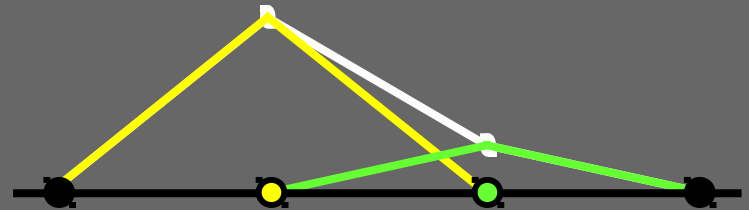
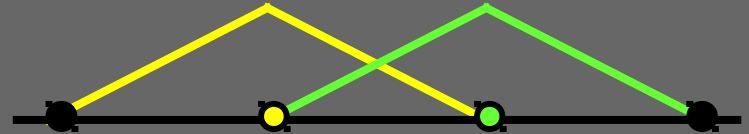
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Discretization

Basis view

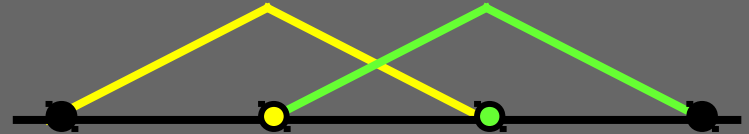
- linear comb. of functions



Discretization

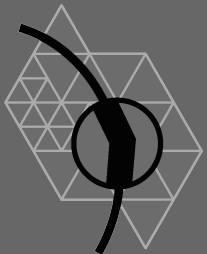
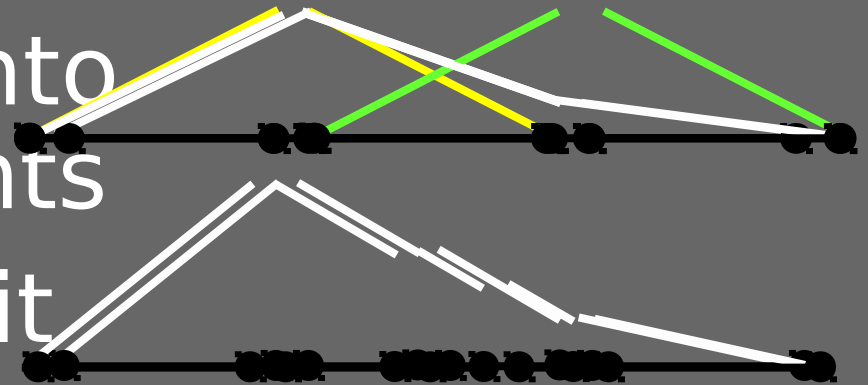
Basis view

- linear comb. of functions



Element view

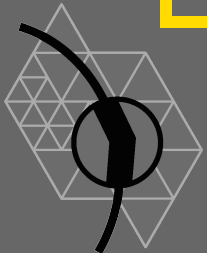
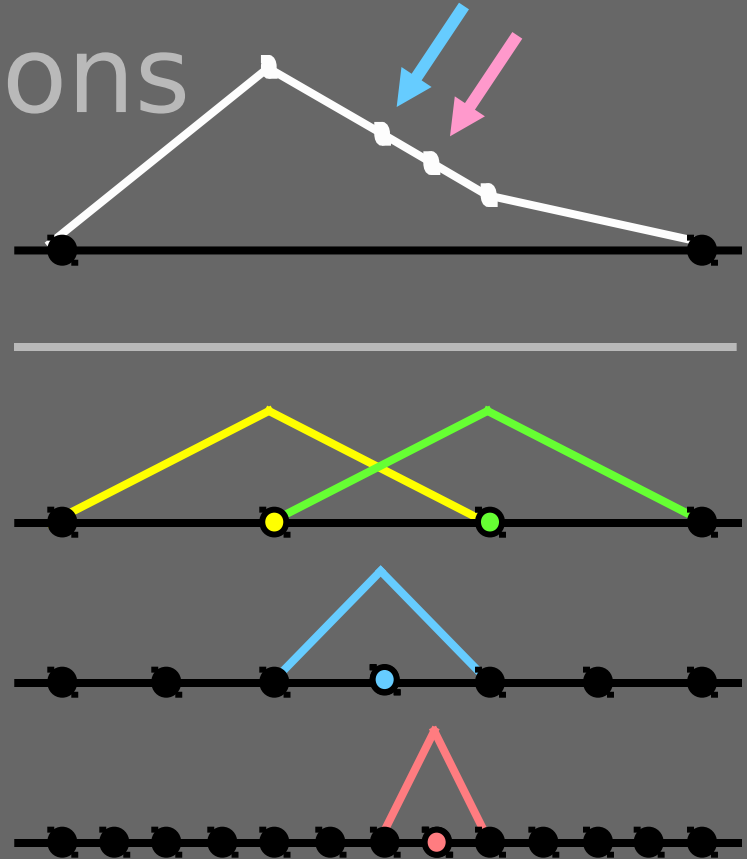
- restriction onto finite elements
- to refine: split



Basis Refinement

Add finer functions

- hierarchy
- linear comb. across levels



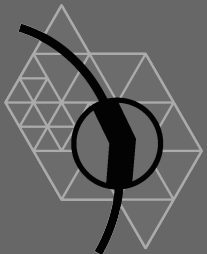
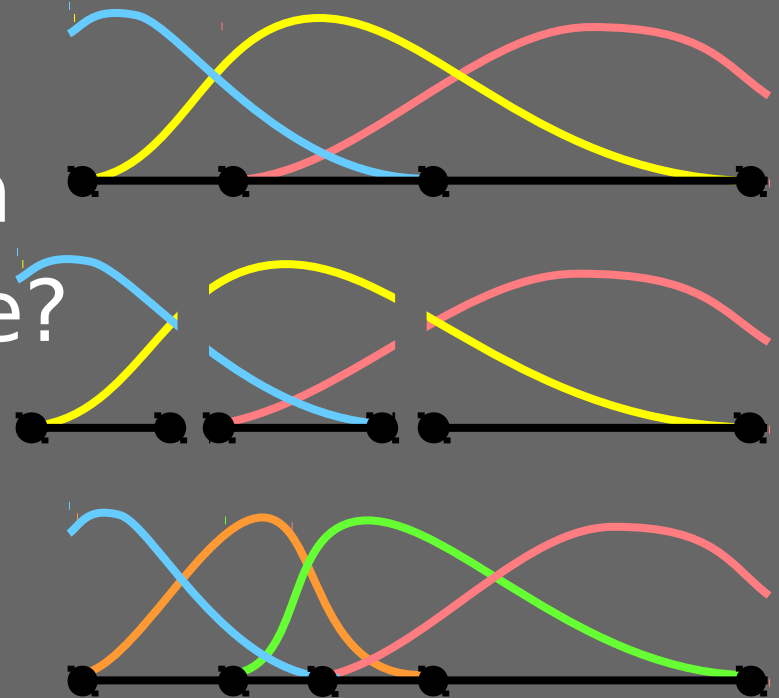
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Element View: Trouble

Higher order schemes

- b-splines, etc.
- ~~knot insertion~~
- bivariate case?



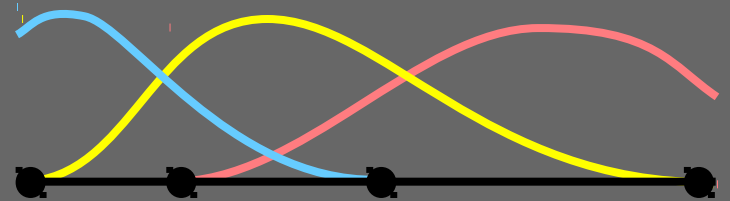
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Element View: Trouble

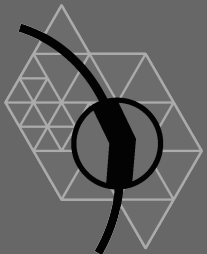
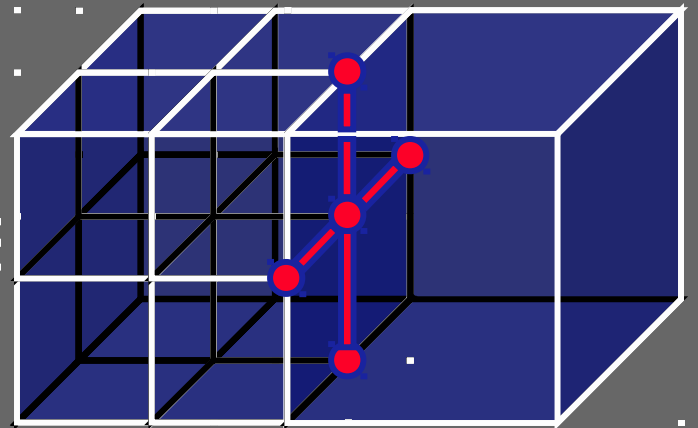
Higher order schemes

- b-splines, etc.
- ~~knot insertion~~
- bivariate case?



2D, 3D, N-D

- incompatibilities
- cumbersome



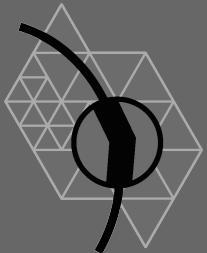
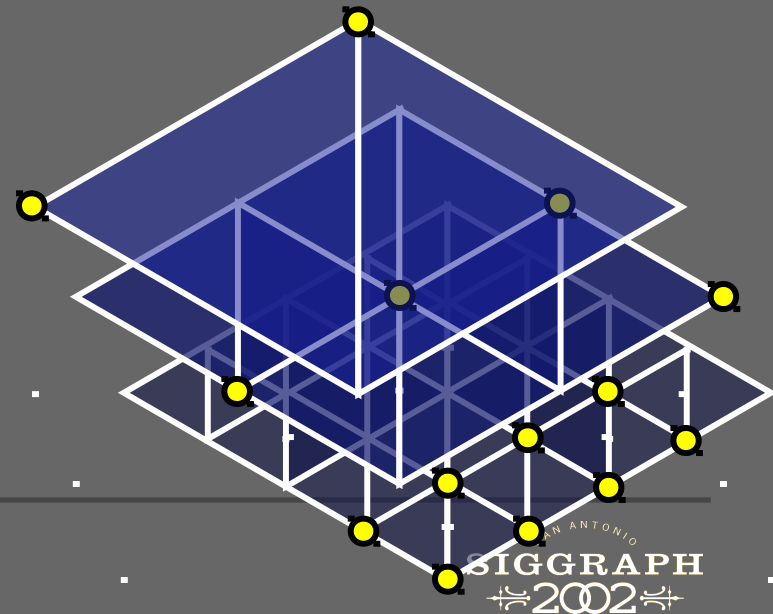
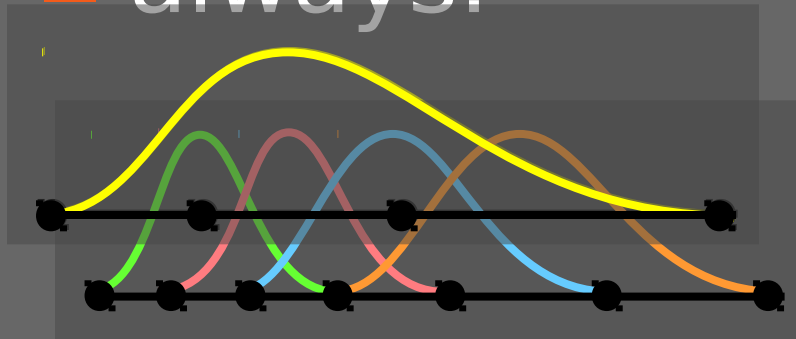
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Basis View: Easy

N-D, high order: business as usual

- naturally compatible
- always:

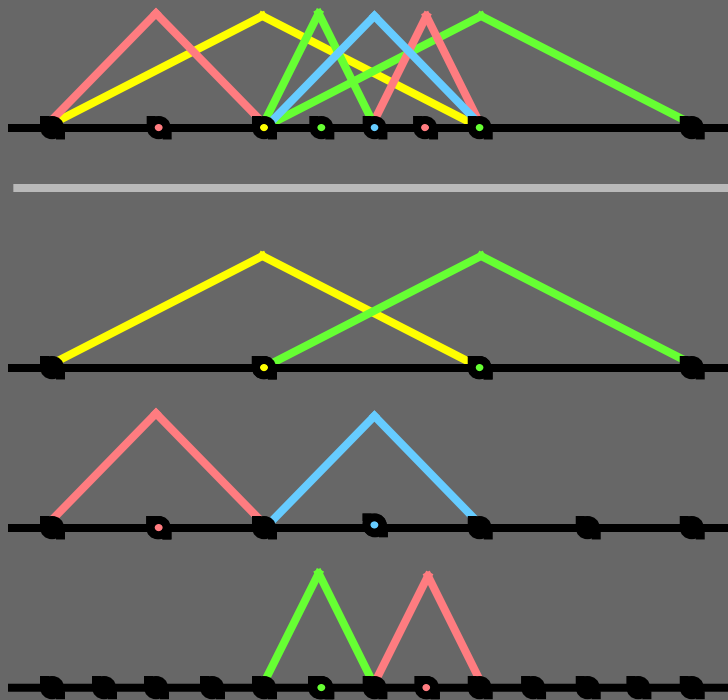


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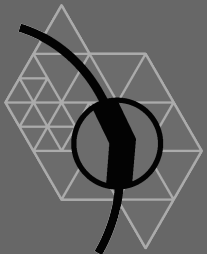
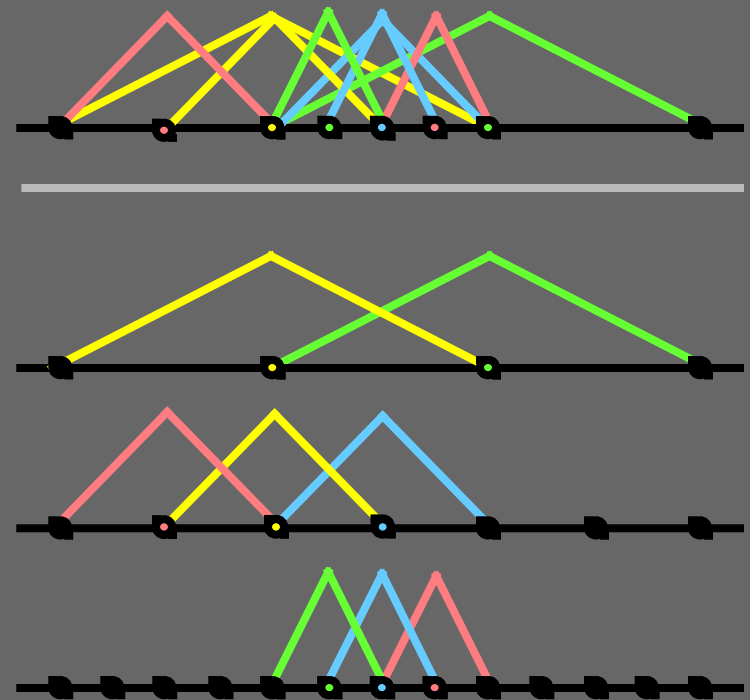
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Refinement Strategies

Augment



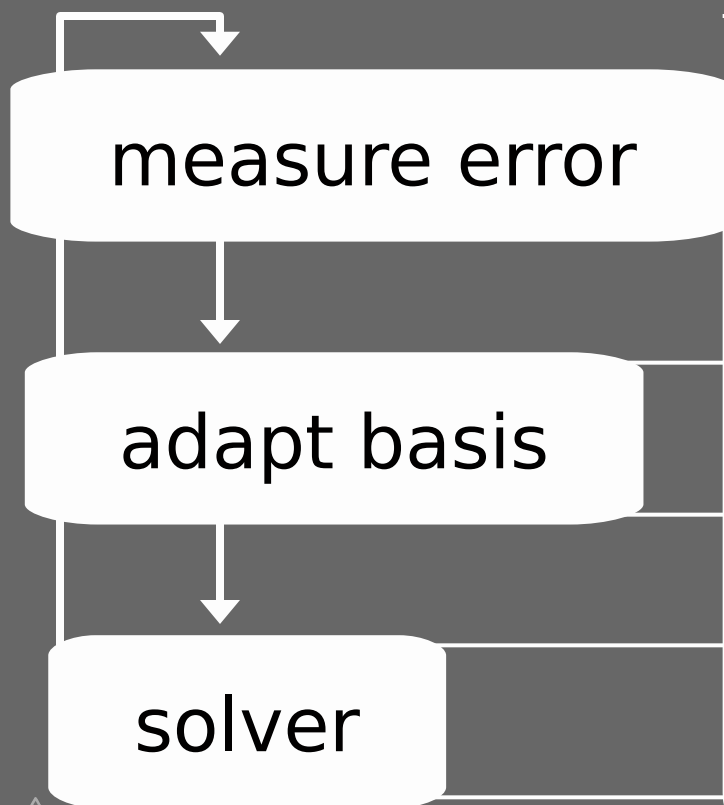
Replace



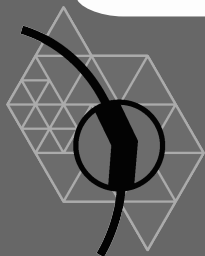
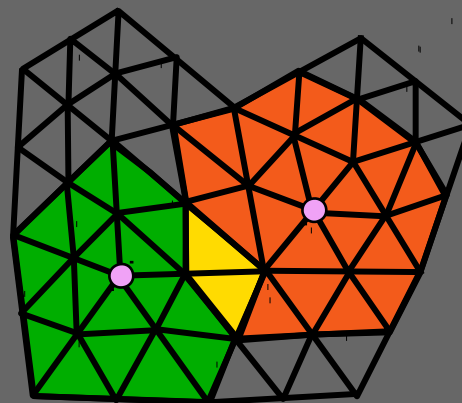
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Framework



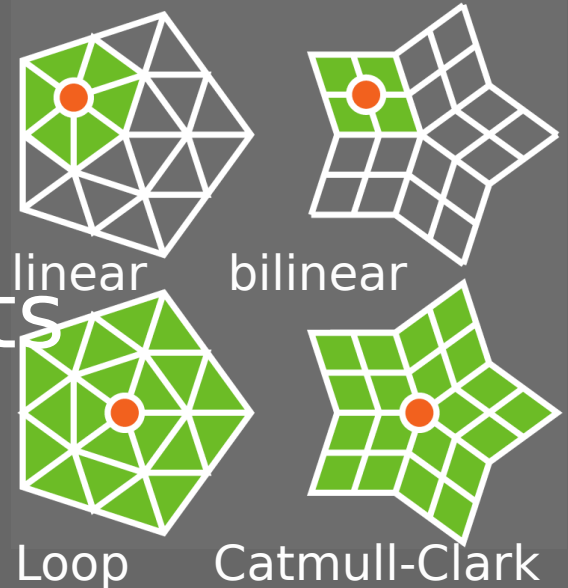
- manage quadratures
- tessellation
- function overlap



Definitions

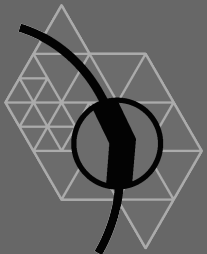
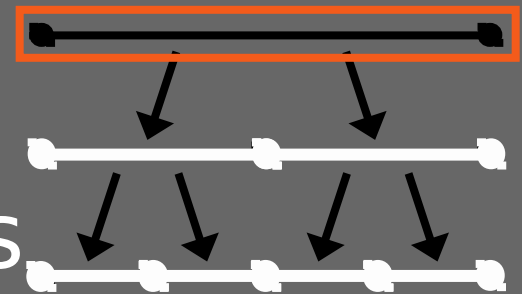
Natural support $\mathcal{S}(\phi)$

- set of overlapping same-level elements



Descendant $\mathcal{D}(e)$

- set of overlapping finer-level elements



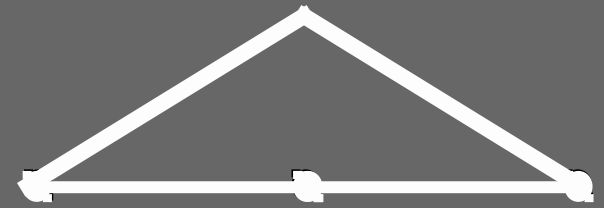
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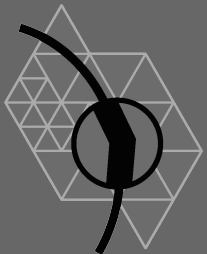
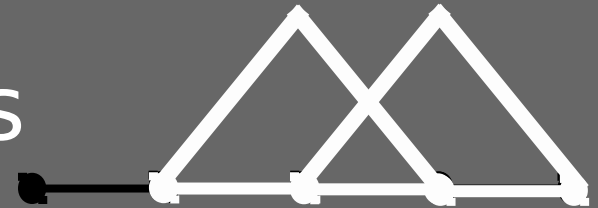
Data Structures

Data structures with invariants

- active functions
- active elements



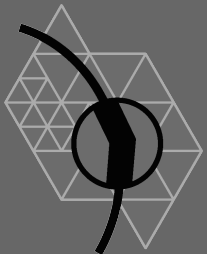
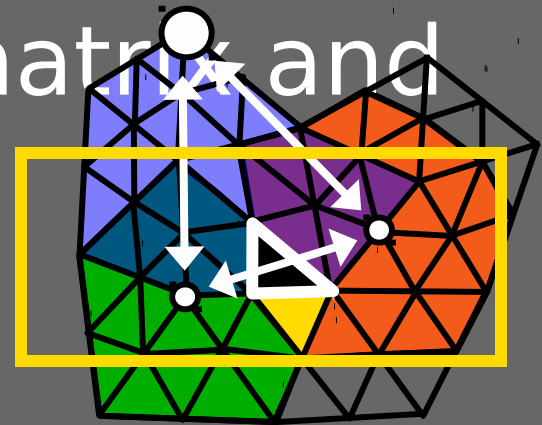
- local overlap tables
 - which functions overlap this element?



Stiffness Matrix

To build stiffness matrix:

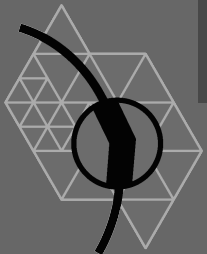
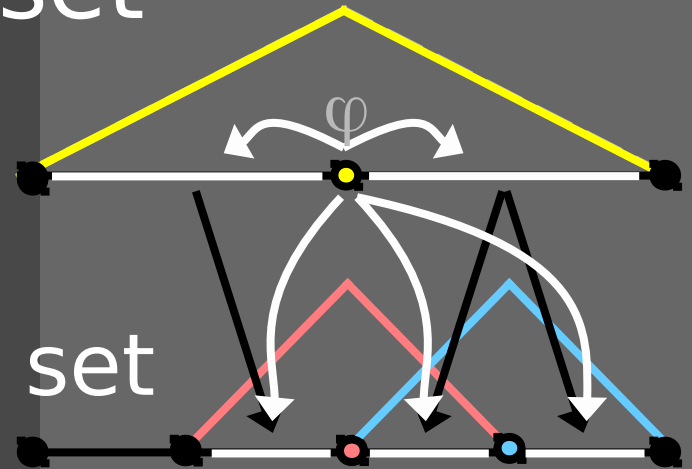
- on every active element:
 - evaluate operator on function pairs
- solver sees only matrix and DOFs



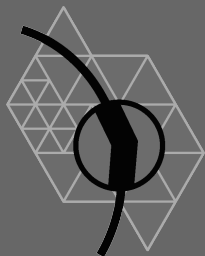
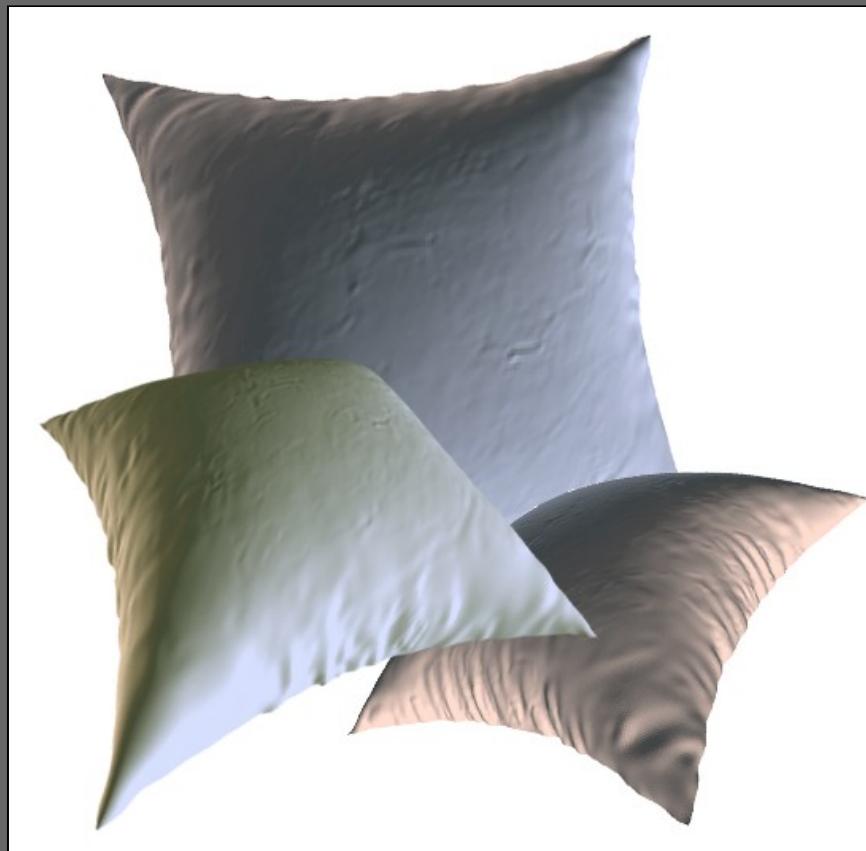
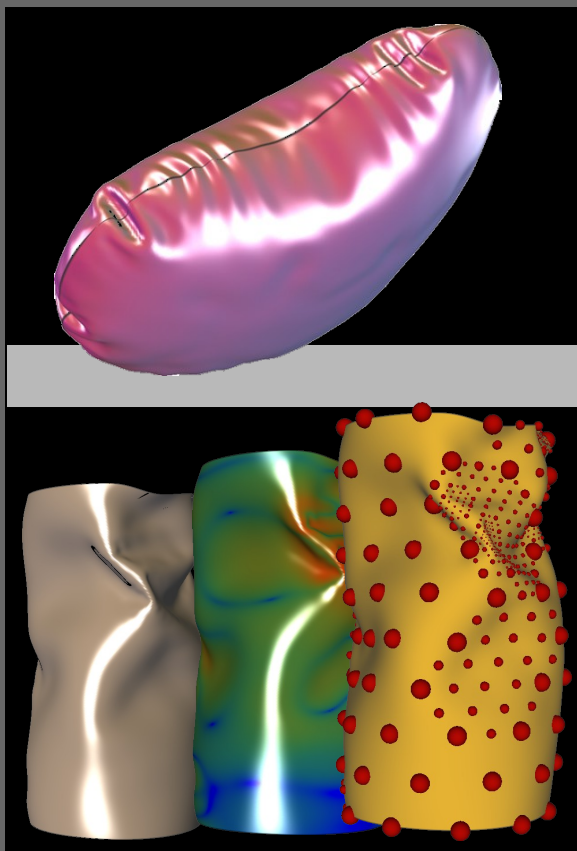
Activation

To activate function φ :

- add φ to active set
- for $e \in S(\varphi)$
 - activate
 - update overlap set
- for $d \in D(e)$
 - update overlap set



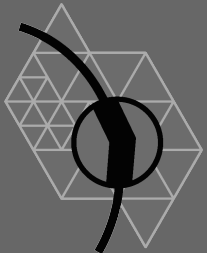
Thin-Shells



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Thin-Shells [MOVIE]

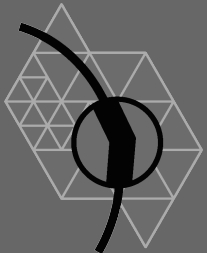
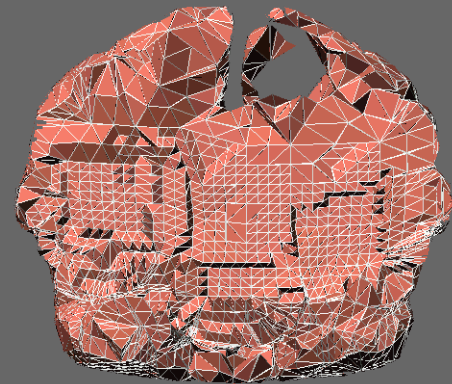
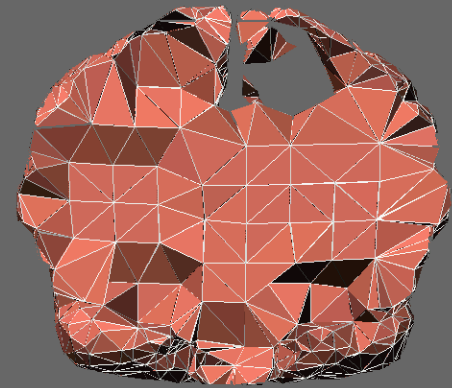
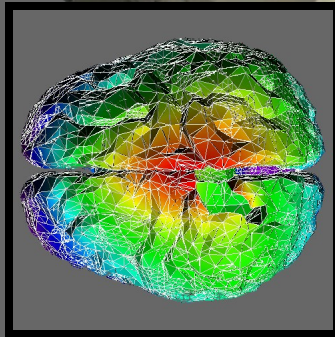
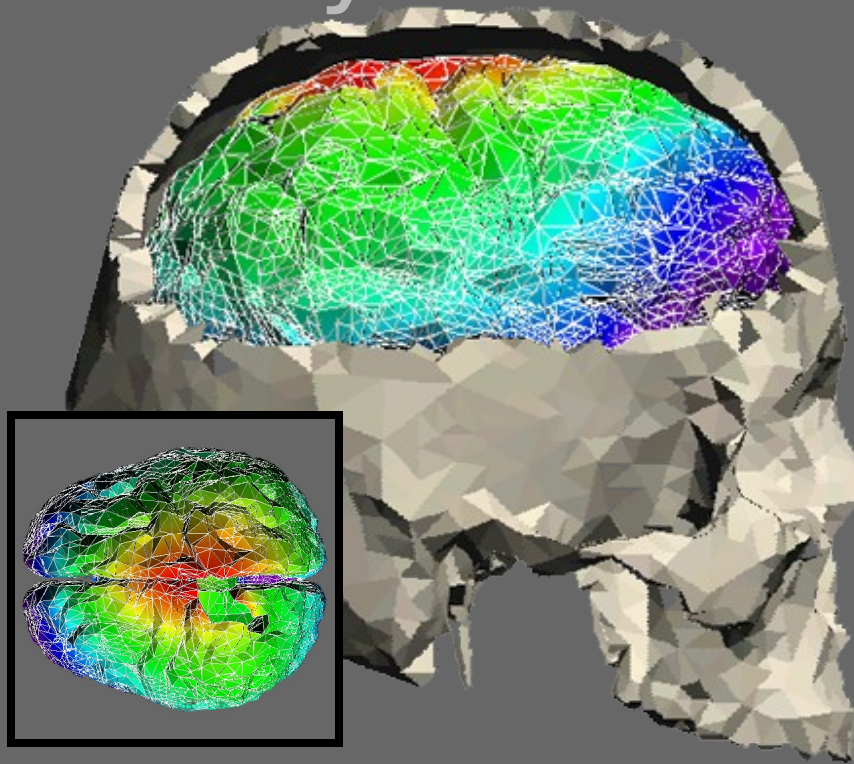


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Surgery Simulation

Elasticity

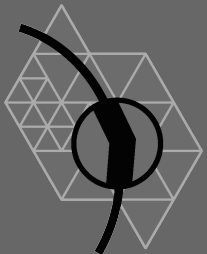
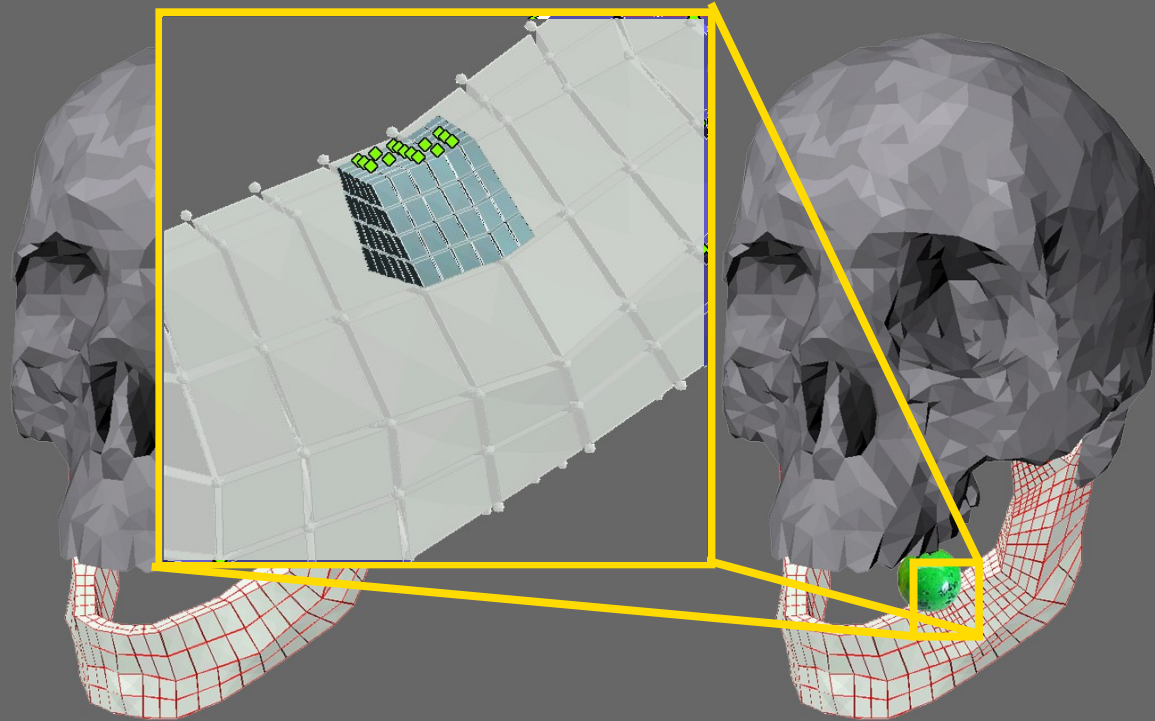


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Biomechanics

Hexahedral elements



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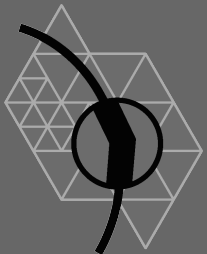
Conclusion

Adaptivity is easy!

- think basis functions, not elements
- naturally compatible
- debug in 1D: easy port to 2D, 3D

Future

- fluids, surgery simulation
- error criteria, solvers



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